MATH 525a ASSIGNMENT 2
FALL 2016
Prof. Alexander
Due Wednesday September 13.

Chapter 1 #5, 6, 8, 9, 12 and the following problems:

(A) For an infinite set X and all $A \in \mathcal{P}(X)$, definte

$$\mu(A) = \begin{cases} \text{card}(A) & \text{if } A \text{ is finite,} \\ \infty & \text{otherwise.} \end{cases}$$

For which X is μ σ-finite?

(B) In the space $X = [0, 1)$, let $\mathcal{M}$ be the set of all countable disjoint unions $\cup_{i \geq 1}[a_i, b_i)$. Show that $\mathcal{M}$ is not a σ-algebra.

(C) Suppose $\mathcal{E}, \mathcal{F}$ are subsets of $\mathcal{P}(X)$, with $\mathcal{E} \subset \mathcal{F} \subset \sigma(\mathcal{E})$. Show that $\sigma(\mathcal{F}) = \sigma(\mathcal{E})$.

(D) Suppose $\mu_1 \leq \mu_2 \leq \dots$ be measures on $(X, \mathcal{M})$. By monotonicity, the limit $\mu(E) = \lim_n \mu_n(E)$ exists for each $E \in \mathcal{M}$. Show that μ is a measure on $(X, \mathcal{M})$.

HINTS:

(8) Consider properties of the sequences $\{F_k\}$ and $\{G_k\}$, where $F_k = \cap_{n=k}^{\infty} E_n$ and $G_k = \cup_{n=k}^{\infty} E_n$.

(12)(c) For general E, F, G, how are the 3 sets $E \Delta G$, $E \Delta F$ and $F \Delta G$ related to each other? For example, is one always contained in the union, intersection, or complement of the other two?

(D) To show that $\mu(\cup_{n \geq 1} E_n) = \sum_{n \geq 1} \mu(E_n)$, show "$\leq$" and "$\geq$". As a side note, the problem is true without the assumption of monotonicity—we just have to assume the limit always exists. But that's much harder to prove, so you're not asked to!