MATH 425a SAMPLE MIDTERM EXAM 2
Fall 2016
Prof. Alexander

(1)(a) State what it means for a series $\sum_n a_n$ to converge.

(b) Prove the following part of Theorem 4.8: for $f : X \to Y$, if f is continuous then $f^{-1}(V)$ is open in X for every open set V in Y.

(2) These two questions are "yes/no with justification." This means formal proof is not required, just say enough to show you understand the reason.

(a) Suppose $f : \mathbb{R} \to \mathbb{R}$ is continuous and $f(x) = 0$ for all rational numbers x. Is it necessarily true that $f(x) = 0$ for all $x \in \mathbb{R}$?

(b) Suppose the Ratio Test is used to show that the series $\sum_n a_n$ converges to some finite s. Is it possible that some rearrangement $\sum_n a'_n$ converges to a value other than s?

(3) Establish convergence or divergence of the series (a), (b) and (c):

(a) $\sum \dfrac{n}{n^3 - 2}$

(b) $\sum 2^{-\sqrt{\log_2 n}}$ (Note $\log_2 n$ means base 2 logarithm.)

(c) $\sum (-1)^n \dfrac{n}{n+1}$

(d) Find the radius of convergence: $\sum n^2 \left(\frac{2}{n}\right)^n z^n$.

(e) Suppose $a_n > 0$ and $a_n/n \to 0$ as $n \to \infty$. Show that $\sum 1/a_n$ diverges. HINT: Comparison test.

(4) Suppose $g : (0, 1] \to \mathbb{R}$ is uniformly continuous, and $p_n > 0$ with $p_n \to 0$.

(a) Show that $\{g(p_n)\}$ is a Cauchy sequence. HINT: All you need are the relevant definitions.

(b) If also $t_n > 0$ with $t_n \to 0$, show that $|g(p_n) - g(t_n)| \to 0$.

(c) Show that $\lim_{x \to 0} g(x)$ exists. HINT: Recall that by Theorem 4.2, this limit exists and is equal to q, if and only if $g(t_n) \to q$ for all sequences $t_n \to 0$ in $(0, 1]$.

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The exam is again closed-book. You will be asked to prove one or more of the following theorems:

3.25a, 3.28, 3.33, 4.8, 4.14, 4.17.

As in Midterm 1, your proof does not have to be the same as what's in the text, just a correct proof—but don't cite a theorem in your proof that appears in the text AFTER the theorem you're proving.

The following will be included with the exam:

REMINDER LIST–TESTS FOR CONVERGENCE

(1) Cauchy criterion ($\{s_n\}$ must be a Cauchy sequence, where s_n is the nth partial sum)
(2) Comparison test
(3) Cauchy condensation test, 3.27
(4) Root test
(5) Ratio test
(6) Alternating series test
(7) Test given by Theorem 3.42 for series $\sum_n a_n B_n$

It is up to you to know what these tests are, and what kind of series they apply to (nonnegative terms only, monotone terms, etc.)