MATH 425a SAMPLE MIDTERM EXAM 1
Fall 2016
Prof. Alexander

You will be asked to prove one or more of the following theorems on the exam:

1.33cde, 2.12, 2.20, 2.24, 2.27a, 2.34, 2.35.

Your proof does not have to be the same as what's in the text, just a correct proof (but don't cite a theorem in your proof that appears in the text AFTER the theorem you're proving.)

(1)(a) Prove Theorem 2.19: every neighborhood $N_r(p)$ is an open set. (NOTE: In the semester when this sample midterm was given, 2.19 was on the list of theorems to know.)

(b) Direct from the definition: Is $\sqrt{2}$ an interior point of the set I of all irrational numbers, in the metric space $\mathbb{R}$? Why or why not? (No formal proof necessary–just a brief statement.)

(2)(a) Define *open cover* (of a set E.)

(b) In $\mathbb{R}$, give an example of an open cover of $\mathbb{Z}$ with no finite subcover. You don't need a formal proof, but say how you know it has no finite subcover.

(c) Suppose F is an infinite set in a metric space, and every $x \in F$ has a neighborhood N_x containing no other point of F (besides x.) Show that F is not compact.

(3) The *boundary* of a set E, denoted ∂E, is $\partial E = \overline{E} \cap \overline{E^c}$.

(a) Show that if every neighborhood of x contains a point of E and a point of E^c, then $x \in \partial E$. (These point of E and/or E^c might be x itself.)

(b) State the converse of part (a).

(c) *Slightly harder problem.* Prove the converse of part (a).

(4) Suppose A is an infinite set, and there is a sequence $\alpha_1, \alpha_2, \dots$ which contains each element of A at least once.

(a) Show that there is a bijection from A to a subset of $\mathbb{N} = \{1, 2, 3, \dots\}$. HINT: Consider where an element of A first appears in the sequence $\{\alpha_i\}$.

(b) Show that A is countable.