MATH 425a MIDTERM EXAM 2
November 4, 2016
Prof. Alexander

Last Name: ______________________________

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Signature: ______________________________

Problem	Points	Score
1	23	
2	31	
3	21	
4	25	
Total	100	

Notes:

(1) Use the backs of the sheets if you need more room.

(2) If you can't do, say, part (a) of a problem, you can assume it and do parts (b), (c), etc.

(3) The list of convergence tests is on the last page.

(1)((16 points)(a) Prove the following part of Theorem 4.8: for $f : X \to Y$, if $f^{-1}(V)$ is open in X for every open set V in Y, then f is continuous. (Note this is the opposite direction from the sample midterm problem.)

(b) Suppose X, Y are metric spaces, $E \subset X$, $f : E \to Y$, and $p \in E'$. Give the definition of $\lim_{x \to p} f(x) = q$.

(2)(31 points)(a) Find the radius of convergence for $\sum(-1)^n \frac{2.5^n}{n^{2.5}} z^n$.

(b) If $a_n \to 0$ and $p > 1$ show that $\sum \frac{a_n}{n^p}$ converges. (We don't assume $a_n \geq 0$.)

Establish convergence or divergence of the following 3 series:

(c) $\sum \frac{1}{4^n - 3^n}$

(d) $\sum_{n \geq 1} \frac{\sin(n) - \sin(n-1)}{n}$. HINT: This is not an alternating series. Consider properties of $\sum_n (\sin(n) - \sin(n-1))$. No trig identities, or computations with trig functions, are needed.

(e) $\sum \left(\frac{2}{3}\right)^{\log_2 n}$. (Here the 2 means base-2 log.)

(3)(21 points) Two unrelated short problems:

(a) Suppose $\{p_n\}$ is a sequence in F,and $\overline{F}$ is compact. Show that $\{p_n\}$ has a Cauchy subsequence.

(b) Suppose $f : X \to Y$ is a one-to-one continuous function, $\{x_n\}$ is a sequence of distinct points in X and x is a limit point of the set $\{x_n : n \geq 1\}$. Show that $f(x)$ is a limit point of $\{f(x_n) : n \geq 1\}$.

(4)(25 points)(a) A function $g : \mathbb{R} \to \mathbb{R}$ is said to satisfy a *Lipschitz condition* if there exist $C, \alpha > 0$ such that $|g(x) - g(y)| \leq C|x - y|^\alpha$ for all x, y. Show that such a g is uniformly continuous.

(b) Define $E = [-1, 0) \cup (0, 1]$ and define $f : E \to \mathbb{R}$ by

$$f(x) = \begin{cases} x + 3, & \text{if } x \in [-1, 0) \\ x + 1, & \text{if } x \in (0, 1] \end{cases}$$

Is f continuous on E? Is f uniformly continuous on E? Prove your answer for "uniformly continuous." For "continuous" you need only give a brief informal justification. HINT: The domain of f is important.

REMINDER LIST–TESTS FOR CONVERGENCE

(1) Cauchy criterion ($\{s_n\}$ must be a Cauchy sequence, where s_n is the nth partial sum)
(2) Comparison test
(3) Cauchy condensation test, 3.27
(4) Root test
(5) Ratio test
(6) Alternating series test
(7) Test given by Theorem 3.42 for series $\sum_n a_n B_n$