MATH 425a MIDTERM EXAM 1 SOLUTIONS
Fall 2016
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(1)(a) (See text)

(b) E is bounded if there exist $p \in X$ and $M > 0$ such that $d(x, p) \leq M$ for all $x \in E$.

(2)(a)(ii) must be true: G^c and K are closed, so $G^c \cap K$ is closed. (iii) must be true: $G^c \cap K$ is a closed subset of the compact set K, so $G^c \cap K$ is compact.

(b) $E \cap G^c$ is an infinite subset of the compact set $G^c \cap K$, so it has a limit point in $G^c \cap K$, that is, a limit point outside G.

(3)(a) C is countable if there exists a bijection between C and the positive integers.

(b) $B^c = (\cap_{n \geq 1} A_n)^c = \cup_{n \geq 1} A_n^c$ is a countable union of countable sets so is countable. If B were also countable then $B \cup B^c = \mathbb{R}$ would be countable, a contradiction. Hence B is uncountable.

(4)(a) $p \in E$ is interior if there exists a neighborhood $N_r(p) \subset E$.

(b) Let $x \in A^\circ \cap B^\circ$. Then $x \in A^\circ$ and $x \in B^\circ$, so there exist neighborhoods $N_{r_1}(x) \subset A$ and $N_{r_2}(x) \subset B$. Let $r = \min(r_1, r_2)$, then $N_r(x) \subset A$ and $N_r(x) \subset B$, so $N_r(x) \subset A \cap B$. This shows $x \in (A \cap B)^\circ$.

(c) Take intervals collapsing to a single point, for example $A_j = (-1/j, 1/j)$. Here A_j is open so $A_j = A_j^\circ$. Then $\cap_{j \geq 1} A_j = \{0\} = \cap_{j \geq 1} A_j^\circ$, but $(\cap_{j \geq 1} A_j)^\circ =$ interior of $\{0\} = \emptyset$. Thus $\cap_{j \geq 1} A_j^\circ = \{0\} \neq \emptyset = (\cap_{j \geq 1} A_j)^\circ$.