MATH 425a MIDTERM EXAM 1
September 28, 2016
Prof. Alexander

Last Name: ______________________________

First Name: ______________________________

USC ID: ______________________________

Signature: ______________________________

Problem	Points	Score
1	23	
2	26	
3	22	
4	29	
Total	100	

Notes:

(1) Use the backs of the sheets if you need more room.

(2) If you can't do, say, part (a) of a problem, you can assume it and do parts (b), (c), etc.

(1)(23 points)(a) Prove Theorem 1.33e: for all complex numbers z, w, we have $|z + w| \leq |z| + |w|$.

(b) For a subset E of a metric space X, state what it means for E to be *bounded*.

(2)(26 points) Suppose K is compact, $G \subset K$, and G is open.

(a) Which of the following properties must hold for $G^c \cap K$? (More than one may be correct.) For the property(s) that must hold, say how you know in a sentence or two; full formal proof not required. For the property(s) that do not have to hold, you need not give justification.

(i) open

(ii) closed

(iii) compact.

(b) Suppose $E \subset K$ with infinitely many points of E outside G. Show that E has a limit point outside G. HINT: What property from (a) is relevant?

(3)(22 points)(a) State what it means for a set C to be *countable.* Give your definition in words only (except for the name C of the set), no mathematical notation.

(b)(15 pts) Suppose $A_1, A_2, ...$ are subsets of $\mathbb{R}$, and for each n the complement A_n^c is countable. Show that $B = \bigcap_{n\geq 1} A_n$ is uncountable.

(4)(29 points)(a) State what it means for a point p to be an *interior point* of a set E.

(b) Show that for sets A, B in a metric space, $A^\circ \cap B^\circ \subset (A \cap B)^\circ$. (Here A° denotes the interior of A.)

(c) Consider the corresponding statement for an infinite sequence of sets: $\cap_{i \geq 1} A_i^\circ \subset (\cap_{i \geq 1} A_i)^\circ$. Give an example to show this can be false. HINT: Nested intervals in $\mathbb{R}$ can work.