2016 Southern California Probability Symposium Abstracts

Georg Menz, UCLA
Variational principles for discrete maps

Previous works have shown that arctic circle phenomenona and limiting behaviors of some integrable discrete systems can be explained by a variational principle. In this talk we will present the first results of the same type for a non-integrable discrete system: graph homomorphisms form Z^d to a regular tree. We will also explain how the technique used could be applied to other non-integrable models. Joint work with Martin Tassy.

Ruth J. Williams, UCSD
Reflected Diffusions and (Bio)Chemical Reaction Networks

Joint work with Saul Leite, Federal University of Juiz de Fora, Brazil
David Anderson, U. Wisconsin-Madison
Des Higham, U. Strathclyde

Continuous-time Markov chain models are often used to describe the stochastic dynamics of networks of reacting chemical species, especially in the growing field of systems biology. These Markov chain models are often studied by simulating sample paths in order to generate Monte-Carlo estimates. However, discrete-event stochastic simulation of these models rapidly becomes computationally intensive. Consequently, more tractable diffusion approximations are commonly used in numerical computation, even for modest-sized networks. However, existing approximations either do not respect the constraint that chemical concentrations are never negative (van Kampen/linear noise approximation) or are typically only valid until the concentration of some chemical species first becomes zero (Langevin approximation).

In this paper, we propose an approximation for such Markov chains via reflected diffusion processes that respect the fact that concentrations of chemical species are never negative. We call this a constrained Langevin approximation because it behaves like the Langevin approximation in the interior of the positive orthant, to which it is constrained by instantaneous reflection at the boundary of the orthant. An additional advantage of our approximation is that it can be written down immediately from the chemical reactions. This contrasts with the van Kampen approximation, which involves a two-stage procedure - first solve a deterministic reaction rate ordinary differential equation, followed by a stochastic differential equation for fluctuations around those solutions. Our approximation also captures the interaction of non-linearities in the reaction rate function with the driving noise. In simulations, we have found the computational cost of our approximation to be at least comparable to, and often better than, that for the van Kampen approximation.

Under mild assumptions, we first prove that our proposed approximation is well defined for all time. Then we prove that it can be obtained as the weak limit of a sequence of jump-diffusion processes that behave like the Langevin approximation in the interior of the positive orthant and

like a rescaled version of the Markov chain on the boundary of the orthant. For this limit theorem, we adapt an invariance principle for reflected diffusions, due to Kang and Williams, and modify a result on pathwise uniqueness for reflected diffusions due to Dupuis and Ishii. Some numerical examples illustrate the advantages of our approximation over direct simulation of the Markov chain or use of the van Kampen approximation.

Tom Trogdon, UCI
Universality in numerical computations with random data

This talk will concern recent progress on the statistical analysis of numerical algorithms with random initial data. In particular, with appropriate randomness, the fluctuations of the iteration count (halting time) of numerous numerical algorithms have been demonstrated to be universal, i.e., independent of the distribution on the initial data. This phenomenon has given new insights into random matrix theory. Furthermore, recent estimates from random matrix theory allow for fluctuation limit theorems for simple algorithms and halting time estimates for others. This is joint work with P. Deift and G. Menon.

Nils Detering, UCSB
Bootstrap Percolation in Directed Inhomogeneous Random Graphs

Bootstrap percolation is a process that is used to model the spread of an infection on a given graph. In the model considered here each vertex is equipped with an individual threshold. As soon as the number of infected neighbors exceeds that threshold, the vertex gets infected as well and remains so forever. We perform a thorough analysis of bootstrap percolation on a novel model of directed and inhomogeneous random graphs, where the distribution of the edges is specified by assigning two distinct weights to each vertex, describing the tendency of it to receive edges from or to send edges to other vertices. Under the assumption that the limiting degree distribution of the graph is integrable we determine the typical fraction of infected vertices. Our model allows us to study a variety of settings, in particular the prominent case in which the degree distribution has an unbounded variance. Among other results, we quantify the notion of ``systemic risk'', that is, to what extent local adverse shocks can propagate to large parts of the graph through a cascade, and discover novel features that make graphs prone/resilient to initially small infections. Further we address the question how systemic risk can be actively managed given the skeleton of the graph.

Yuval Peres, Microsoft Research
Pinpointing mixing time in expanders and random graphs

We determine precisely the mixing time of random walk on optimal d-regular expander graphs (also known as Ramanujan graphs.) and show it is the time it takes the walk to reach the typical distance from its starting point. These walks exhibit **cutoff:** a sharp decrease of the total variation distance to the stationary measure in a relatively short time interval. On the other hand, we show that for random walks on the giant component of supercritical Erdos-Renyi random graphs,

mixing occurs long after the typical distance from the starting point is reached; this is explained via a dimension drop for harmonic measure on Galton-Watson trees, discovered with Lyons and Pemantle (1995). Nevertheless, from most starting nodes, the cutoff phenomenon holds. Talk based on joint works with Eyal Lubetzky (GAFA 2016, http://arxiv.org/abs/1507.04725) and with Berestycki-Lubetzky-Sly (http://arxiv.org/abs/1504.01999).