Conditional evenly convex sets and the representation of conditional maps.

Marco Frittelli[*]

We introduce the concept of *conditional* evenly convex sets, as the generalization of the "static" notion of evenly convex set, and show that it is tailor made for the representation of conditional maps, as for example conditional quasi-convex risk measures.

In the classical approach, the conditional maps $\rho : L^p(\Omega, \mathcal{F}_T, \mathbb{P}) \to L^p(\Omega, \mathcal{F}_t, \mathbb{P})$, $t < T$, are defined on vector spaces. Here instead we work in a random locally convex module. The intuition behind the use of modules is simple and natural: Consider an agent that is computing the risk of a time T portfolio at an intermediate time $t < T$. Thus the $\mathcal{F}_t$ measurable random variables will act as scalars in the process of diversification of the portfolio, forcing to consider the set

$$\begin{aligned} L^p_{\mathcal{F}_t}(\mathcal{F}_T) \quad &: \quad = L^0(\Omega, \mathcal{F}_t, \mathbb{P}) \cdot L^p(\Omega, \mathcal{F}_T, \mathbb{P}) \\ &= \quad \left\{ YX \mid Y \in L^0(\Omega, \mathcal{F}_t, \mathbb{P}),\ X \in L^p(\Omega, \mathcal{F}_T, \mathbb{P}) \right\} \end{aligned}$$

as the domain of the conditional maps under consideration.

The use of the module approach and the notion of conditional evenly convex sets allow us to obtain a complete duality for quasi-convex monotone conditional maps. Joint paper with Marco Maggis

[*]Dipartimento di Matematica, Università degli Studi di Milano.